Transportation 7

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Transportation techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Recognise a problem of Transportation.
- State a Transportation problem in the form of a Linear Programming problem.
- Find Initial Basic Feasible Solution by various methods.
- Find minimum Transportation Cost schedule.
- Obtain solutions for the Special cases of Transportation like Unbalanced Problem, Maximisation Problem, Problem having Multiple Optimum Solution, Problem having Degeneracy, Restricted Transportation Problem etc.

Transportation

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Transportation problem is a special kind of Linear Programming Problem, in which goods are transported from a set of Sources to a set of Destinations, subject to the constraints of supply and demand such that the total Cost of Transportation is minimized. It is also sometimes called Hitchcock problem because it was originally developed by F. L. Hitchcock in his study titled " The distribution of a product from several sources to numerous locations" in the year 1941. Subsequently in the year 1947, T. C. Koopmans independently published a study on "Optimum utilization of the Transportation System". The linear programming formulation and the associated systematic procedure for solution was given by George B. Dantzig afterwards – in the year 1951.

The easiest way to recognise a Transportation problem is to consider a typical situation as shown in the figure below.

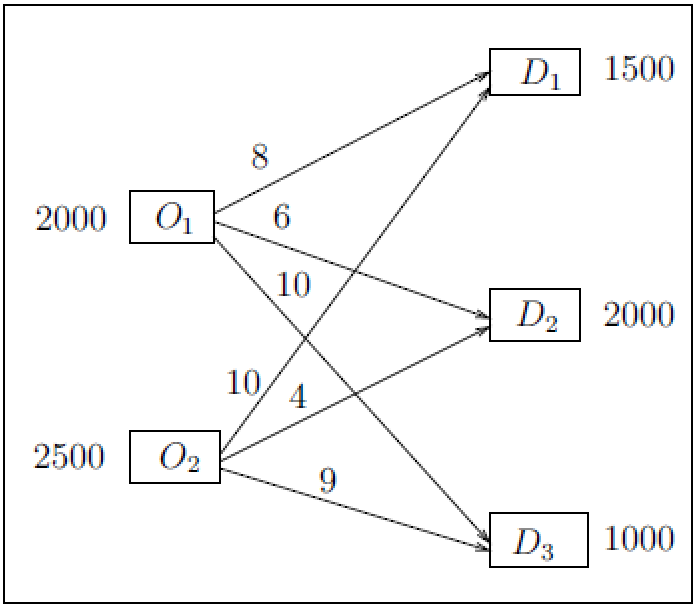

Figure 7.1: Typical Situation of Transportion problem

Assume that 2 Bread Factories O_1 and O_2 make the daily bread in a city. The bread produced is supplied to the three Bakeries of the City – D_1, D_2 and D_3. The production capacities of the Bread Factories are 2000 & 2500 units respectively and demands of the Bakeries are 1500, 2000 and 1000 units as shown in the diagram above. Also shown are the Transportation Costs per unit of bread along different routes. The problem is to determine the quantity each Factory should transport to each Bakery so that the Total Cost of Transportation is minimum.

Linear Programming formulation of Transportation Problem

With reference to the problem mentioned above, following decision variables are defined.

x_{ij} = Number of units of bread to be delivered by the Factory O_i to the Bakery D_j where i = 1, 2 & j = 1, 2, 3

Corresponding model of LPP is given as

Minimize $Z = 8x_{11} + 6x_{12} + 10x_{13} + 10x_{21} + 4x_{22} + 9x_{23}$

Subject to the constraints

$x_{11} + x_{12} + x_{13} = 2000$ (Supply constraint of Factory O_1)

$x_{21} + x_{22} + x_{23} = 2500$ (Supply constraint of Factory O_2)

$x_{11} + x_{21} = 1500$ (Demand constraint of Bakery D_1)

$x_{12} + x_{22} = 2000$ (Demand constraint of Bakery D_2)

$x_{13} + x_{23} = 1000$ (Demand constraint of Bakery D_3)

$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}$ & $x_{23} \geq 0$ (Non negativity constraint)

Also one more condition is there and that is, Total Supply = Total Demand

The generalised form of the problem can be written as follows –

$$\min Z = \sum_{i=1}^{m} \sum_{i=1}^{n} C_{ij} X_{ij}$$

Subject to the constraints –

$$\sum_{j=1}^{n} x_{ij} = a_i$$

$$\sum_{i=1}^{m} x_{ij} = b_j$$

and $x_{ij} \geq 0$,

where i = 1, 2, ... m and j = 1, 2, ... n

The problem has a feasible solution only if the total capacity of the Sources and total demand of the Destinations are equal. This is known as Rim Condition.In mathematical terms that can be written as follows –

$$\sum_{j=1}^{n} a_j = \sum_{i=1}^{m} b_i = \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij}$$

The term **Feasible Solution** mentioned above refers to a set of non-negative values of x_{ij} (where i = 1, 2, ... m and j = 1, 2, ... n) those satisfy all the constraints. An initial feasible solution with (m + n – 1) number of values of the variable is called **Basic Feasible Solution**.

Underlying **assumptions** of a Transportation problem are –

A. Only a single type of commodity is being shipped from different Sources to various Destinations.

B. Total supply = Total demand

C. The unit Transportation cost of the item from all sources to different destinations are certainly and precisely known.

D. The objective is to minimize the total cost of transportation.

Procedure of solution of Transportation Problems

Step - (A): Obtain a Basic Feasible Solution.

Step - (B): Test the solution obtained in Step (A) for optimality

Step - (C): If the operation of Step (B) suggests for a non-optimal Solution then go to Step - (D)

Step - (D): Improve the basic feasible solution.

Step - (E): Repeat Steps (B), (C) & (D) until optimal solution is obtained.

A. Obtaining Basic Feasible Solution

The following are the methods for obtaining Basic Feasible Solution of transportation problem:

1. North West Corner Method (NWCM)
2. Least Cost Method (LCM)
3. Vogel's Approximation Method (VAM)

1. North West Corner Method (NWCM):

This is the simplest of all the methods mentioned above for obtaining Basic Feasible Solution of a Transportation problem. The name of the method is derived from the fact that the solution begins by allocating maximum possible quantity to the north-west or top left corner cell of the Transportation matrix given. Step by step methodology of solution is given below:

1. Allocate as many units as possible in the north-west or left top corner cell of the Transportation matrix by maintaining supply and demand constraint corresponding to this cell. In fact minimum of the value of supply or demand corresponding to the mentioned cell should be allocated.
2. Subtract the quantity allocated in the previous step from the supply and demand figures in the first row and the column to get the adjusted figures.
3. Depending on which of the values of supply or demand exhausted fully in step before, follow either one of the steps mentioned below -
   (a) If the supply for the first row is exhausted then move down to the first cell of the second row & first column and once again allocate as maximum as possible quantity maintaining the constraint of supply and demand. Subsequently adjust the quantities of supply and demand by subtracting the quantity allocated from them.
   (b) If the demand for the first column is exhausted then move sideways to the next cell in the second column and first row and as before allocate as maximum as possible quantity in the cell. Subsequently adjust the corresponding figures of supply and demand.
4. Continue the procedure until total available quantity is fully allocated to the cells as required. *In case for any* cell the supply and demand both exhausts at a time (in other words supply = demand for the cell) then next allocation can be made in the cell either in the next row or column.

The quantities so allocated should be **circled** to indicate the value of the corresponding variable.

[**Note** – Though the method is very easy to compute, understand and interpret, the greatest drawback is the fact that transportation costs are not given any consideration. As a result, the Initial Feasible Solution obtained by this method is mostly non-optimal. Usually several iterations are required before an optimal solution is reached.]

Illustration 1

A company dealing with a special type of liquid has three plants P_1, P_2 & P_3 located throughout the country. Production capacities of these plants are respectively 50, 75 and 25 Gallons. Each day the firm must furnish to four of its Retail shops R_1, R_2, R_3, and R_4 with at least 20, 20, 50 and 60 Gallons of the product. The transportation cost per Gallon (in '000 ₹) between various sets of Plants and Retail Shops are given below –

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	20	20	50	60	

Find the Basic Feasible Solution for the problem of transportation using NWCM.

Solution:

Starting from North West corner cell, min. between 50 & 20 (i.e. 20) is allocated to the cell P_1R_1. It is shown below

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	(20) 3	5	7	6	30 ~~50~~
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	20	50	60	

From the Table above it is clear that the demand of the Retail Shop (R_1) represented along 1st column is completely satisfied and we shift sideways to cell P_1R_2 for next allocation. Maintaining the constraint between Supply and Demand here also 20 Gallons allocation is possible. As a result demand of the 2nd column is also exhausted completely. The table below shows this allocation along with the previous one.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3 (20)	5 (20)	7	6	~~30~~ 10 ~~50~~
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	~~20~~	50	60	

Again we move horizontally to the cell P_1R_3 and allocate 10 Gallons which causes complete exhaustion of Supply from P_1. In other words Supply corresponding to the 1st row is totally exhausted. It is shown in the table below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3 (20)	5 (20)	7 (10)	6	~~30~~ ~~10~~ ~~50~~
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	~~20~~	~~50~~ 40	60	

Next allocation of 40 Gallons is done in cell P_2R_3 and the resultant table is shown below. It is clear that the Demand of R_3 is fully meet up due to this allocation. In other words Column R_3 is totally exhausted now.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3 (20)	5 (20)	7 (10)	6	~~30~~ ~~10~~ ~~50~~
P_2	2	5	8 (40)	2	35 ~~75~~
P_3	3	6	9	2	25
Demand	~~20~~	~~20~~	~~50~~ ~~40~~	60	

Thereafter allocation of 35 Gallons is done in cell P_2R_4. This has completely exhausted the Supply capacity of P_2 or the 2nd row, but the 4th column or R_4 still has a demand of 25 Gallons which has to be fulfilled by the Plant P_3. The results are shown in the table below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	(20) 3	(20) 5	(10) 7	6	~~30~~ ~~10~~ ~~50~~
P_2	2	5	(40) 8	(35) 2	~~35~~ ~~75~~
P_3	3	6	9	2	25
Demand	~~20~~	~~20~~	~~50~~ ~~40~~	~~60~~ 25	

The final allocation of 25 Gallons is done in the cell P_3R_4 and the complete set of allocations is shown in the Table below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	(20) 3	(20) 5	(10) 7	6	~~30~~ ~~10~~ ~~50~~
P_2	2	5	(40) 8	(35) 2	~~35~~ ~~75~~
P_3	3	6	9	(25) 2	~~25~~
Demand	~~20~~	~~20~~	~~50~~ ~~40~~	~~60~~ ~~25~~	

The Basic Feasible Solution of the given Transportation problem by NWCM is given as follows –

from Plant	to Retail Shop	Shipping cost per Gallon (₹ '000)	Quantity allocated (Gallons)	Cost (₹ '000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P_1	R_1	3	20	60
P_1	R_2	5	20	100
P_1	R_3	7	10	70
P_2	R_3	8	40	320
P_2	R_4	2	35	70
P_3	R_4	2	25	50
TOTAL				670

Total Cost of Transportation = ₹ 6,70,000

2. Least Cost Method (LCM):

Unlike NWCM, in this method, due consideration is given to the cost. As a result, it reduces the computations while obtaining the optimal solution, that is the minimum total cost of transportation. Step by step methodology of arriving at the Basic Feasible solution according to this method is given as –

1. Select the cell with lowest shipping cost among all the cells of the given Transportation matrix. If such a cell is not unique then select a cell out of all the cells tied with the lowest cost value such that maximum allocation is possible there. In case there is a tie in that, too then arbitrarily choose one among the cells with tied lowest cost as well as maximum possible allocation value.
2. Allocate as maximum as possible units to the cell selected in step (1) by maintaining the Supply and Demand constraint corresponding to the selected cell. As a result either the Supply figures of a row or the Demand figures of a column will be fully exhausted. Such a row or column should be eliminated for finding further allocations.
3. Reduce the Supply value of the Row or the Demand value of the Column (which is not completely exhausted) by the amount allocated in the previous step.
3. Repeat above steps until all the allocations are over.

Illustration 2

Use data of Illustration 1, to find the Basic Feasible Solution by LCM.

Solution:

Out of all the cells of the given Transportation matrix, the lowest cost figure (of 2) appears at three different cells, namely - P_2R_1, P_2R_4 and P_3R_4. Again P_2R_4 can have maximum allocation (of 60 Gallons) amongst the three. So we start the solution by allocating 60 at P_2R_4. The results are shown in the table below

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	2	5	8	2 (60)	15 ~~75~~
P_3	3	6	9	2	25
Demand	20	20	50	~~60~~	

Due to this allocation, total demand of R_4 is fulfilled and 4th column is eliminated from the matrix during further allocations.

Out of the remaining cells, P_2R_1 is having lowest cost (2). Maintaining the constraint of Supply and Demand, we allocate 15 Gallons at this cell. As a result the total supply capacity of P_2 is exhausted and 2nd row is eliminated from further considerations. The result is shown in the table below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	(15) 2	5	8	(60) 2	~~15~~ ~~75~~
P_3	3	6	9	2	25
Demand	~~20~~ 5	20	50	~~60~~	

Out of the remaining cells both P_1R_1 and P_3R_1 have same value of least cost (3) and also both can have an allocation of 5 Gallons which is the Demand yet to be fulfilled. So we **arbitrarily** choose P_3R_1 and allocate 5 Gallons in it. This will completely exhaust the Demand of R_1 and 1st column will no longer be considered for further allocations. The result is shown in the table below

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	(15) 2	5	8	(60) 2	~~15~~ ~~75~~
P_3	(5) 3	6	9	2	~~25~~ 20
Demand	~~20~~ ~~5~~	20	50	~~60~~	

Next we allocate 20 Gallons at the cell P_1R_2 which has the least cost (5) among the remaining cells. Due to this, Demand of R_2 will be completely filled up and column 2 is omitted from further consideration. The resulting table is shown below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	(20) 5	7	6	~~50~~ 30
P_2	(15) 2	5	8	(60) 2	~~15~~ ~~75~~
P_3	(5) 3	6	9	2	~~25~~ 20
Demand	~~20~~ ~~5~~	~~20~~	50	~~60~~	

Remaining two cells P_1R_3 and P_3R_3 are allocated with the remaining quantities of 30 and 20 Gallons respectively.

The resulting matrix is shown below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5 (20)	7 (30)	6	~~50~~ ~~30~~
P_2	2 (15)	5	8	2 (60)	~~15~~ ~~75~~
P_3	3 (5)	6	9 (20)	2	~~25~~ ~~20~~
Demand	~~20~~ ~~5~~	~~20~~	~~50~~ ~~20~~	~~60~~	

The Basic Feasible Solution of the Transportation problem by LCM is given as follows –

from Plant	to Retail Shop	Shipping cost per Gallon (₹ '000)	Quantity allocated (Gallons)	Cost (₹ '000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P_1	R_2	5	20	100
P_1	R_3	7	30	210
P_2	R_1	2	15	30
P_2	R_4	2	60	120
P_3	R_1	3	5	15
P_3	R_3	9	20	180
			TOTAL	655

Total Cost of Transportation = ₹ 655,000

So improvement is there [by an amount of ₹(670000 – 655000) = ₹ 15000] in the solution obtained by LCM over that by NWCM.

3. Vogel's Approximation Method (VAM):

Like LCM, this method also makes use of the Cost figures. Instead of least cost, the difference between the least cost and the value next to least cost is computed for each and every row and column. Such difference is termed as Penalty. Then least penalty value is used as the starting point of allocation. In fact the Basic Feasible Solution obtained by this method is generally Optimal or near Optimal. As a result number of iterations required to arrive at the final set of allocations is much less compared to the previous two methods. So always it is preferred for getting the Basic Feasible Solution. Stepwise methodology of arriving at the solution by VAM is given as follows:

1. Calculate Penalty values for each Row and Column. (where Penalty = Difference between the smallest value and next smallest value of unit transportation costs provided in the problem). In case there are two or more minimum value in the same Row or Column then the corresponding Penalty is zero.

2. Identify the Row or Column having highest of the Penalty values computed in Step (1).
3. In the identified Row or Column choose the cell having least cost figure and allocate as maximum as possible number of units maintaining the constraint of Supply and Demand. As a result either the Supply or the Demand will be fully exhausted and the corresponding Row or the Column should be omitted from consideration of further allocations. In case both Row and Column are fully exhausted, then omit both from further considerations.

   If a TIE occurs in the penalties, select the Row or the Column which has minimum cost figure among its cells. If there is a TIE in that value also then go for the cell which can have highest allocation among such cells. If there is a TIE in this value, too then arbitrarily choose one among such cells.
4. Reduce the Supply value of the Row or the Demand value of the Column (which is not completely exhausted) by the amount allocated in the previous step.
5. Re-compute the Row and Column penalties for the reduced matrix of Transportation.
6. Repeat steps (3) and (4).
7. Repetition of the above steps to be continued until all allocations are done. In other words it should continue until total Supply and Demand values are fully exhausted.

Illustration 3

Use data of Illustration 1, to find the Basic Feasible Solution by VAM.

Solution:

Penalty values for each row and column are calculated and entered in the table as shown. Of all the penalties, the one corresponding to the 1st Row is highest (background marked with a different colour). Corresponding to 1st Row least cost (3) appears in the cell P_1R_1. Maximum possible allocation in this cell is 20 Gallons. Also that exhausts the demand of R_1. Hence Column for R_1 is shaded meaning no further participation of it is possible in the subsequent computations. Results are given in the following table.

from Plants	to Retail Shops				Supply	Row pen. 1
	R_1	R_2	R_3	R_4		
P_1	3 (20)	5	7	6	30 ~~50~~	2*
P_2	2	5	8	2	75	0
P_3	3	6	9	2	25	1
Demand	~~20~~	20	50	60		
Col pen. 1	1	0	1	0		

Row pen. = Row penalty
Col. pen. = Column penalty

Next set of penalties are calculated omitting 1st Column and entered in the table. Of all, that corresponding to 3rd Row is maximum. Again least cost figure in 3rd Row is 2 which corresponds to cell P_3R_4. Maximum possible allocation here is 25 Gallons which exhausts the supply capacity of P_3. It is shaded to indicate its omission from further participation in the new computations. Results are given in the table below.

from Plants	to Retail Shops				Supply	Row pen. 1	Row pen. 2
	R_1	R_2	R_3	R_4			
P_1	3 (20)	5	7	6	30 ~~50~~	2	1
P_2	2	5	8	2	75	0	3
P_3	3	6	9	2 (25)	~~25~~	1	4*
Demand	~~20~~	20	50	~~60~~ 35			
Col pen. 1	1	0	1	0			
Col pen. 2	-	0	1	0			

Row pen. = Row penalty
Col. pen. = Column penalty

Next set of penalties are calculated omitting 1st Column and 3rd Row entered into the table. Of all, that corresponding to 4th Column is maximum. Again least cost figure in 4th Column is 2 which corresponds to cell P_2R_4. Maximum possible allocation here is 35 Gallons which exhausts the demand of R_4. It is shaded to indicate its omission from further participation in the new computations. Results are given in the table below.

from Plants	to Retail Shops				Supply	Row pen. 1	Row pen. 2	Row pen. 3
	R_1	R_2	R_3	R_4				
P_1	3 (20)	5	7	6	30 ~~50~~	2	1	1
P_2	2	5	8	2 (35)	40 ~~75~~	0	3	3
P_3	3	6	9	2 (25)	~~25~~	1	4	-
Demand	~~20~~	20	50	~~60~~ ~~35~~				
Col pen. 1	1	0	1	0				
Col pen. 2	-	0	1	0				
Col pen. 3	-	0	1	4*				

Row pen. & Col. pen. stands for Row & Column penalties respectively.

Next set of penalties are calculated omitting 1st Column , 3rd Row and 4th Column & entered into the table. Of all, that corresponding to 2nd Row is maximum. Again least cost figure in the 2nd Row is 5 which correspond to the cell P_2R_2. Maximum possible allocation here is 20 Gallons which exhausts the demand of R_2. It is shaded to indicate its omission from further participation in the new computations. Results are given in the table below.

from Plants	to Retail Shops R1	R2	R3	R4	Supply	Row pen. 1	Row pen. 2	Row pen. 3	Row pen. 4
P_1	3 (20)	5	7	6	30 ~~50~~	2	1	1	2
P_2	2	5 (20)	8	2 (35)	~~40~~ ~~75~~ 20	0	3	3	3*
P_3	3	6	9	2 (25)	~~25~~	1	4	-	-
Demand	~~20~~	~~20~~	50	~~60~~ ~~55~~					
Col pen. 1	1	0	1	0					
Col pen. 2	-	0	1	0					
Col pen. 3	-	0	1	4					
Col pen. 4	-	0	1	-					

Row pen. & Col. pen. stands for Row & Column penalties respectively.

From above it is clear that the two remaining cells P_1R_3 and P_2R_3 should have allocations of the remaining quantities of 30 and 20 Gallons in the 1st and 2nd Rows respectively. These allocations are done and the final table with all possible allocations is shown below.

from Plants	to Retail Shops R1	R2	R3	R4	Supply	Row pen. 1	Row pen. 2	Row pen. 3	Row pen. 4
P_1	3 (20)	5	7 (30)	6	~~30~~ ~~50~~	2	1	1	2
P_2	2	5 (20)	8 (20)	2 (35)	~~40~~ ~~75~~ ~~20~~	0	3	3	3*
P_3	3	6	9	2 (25)	~~25~~	1	4	-	-
Demand	~~20~~	~~20~~	~~50~~	~~60~~ ~~55~~					
Col pen. 1	1	0	1	0					
Col pen. 2	-	0	1	0					
Col pen. 3	-	0	1	4					
Col pen. 4	-	0	1	-					

Row pen. & Col. pen. stands for Row & Column penalties respectively.

The Basic Feasible Solution of the Transportation problem by VAM is given as follows –

from Plant	to Retail Shop	Shipping cost per Gallon (₹000)	Quantity allocated (Gallons)	Cost (₹000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P_1	R_1	3	20	60
P_1	R_3	7	30	210
P_2	R_2	5	20	100
P_2	R_4	2	35	70
P_2	R_3	8	20	160
P_3	R_4	2	25	50
			TOTAL	650

Total Cost of Transportation = ₹ 650,000/-

So improvement is there [by an amount of ₹ (670000 – 650000) = ₹ 20000/-] in the solution obtained by VAM over that by NWCM and also [by an amount of ₹ (655000 – 650000) = ₹ 5000/-] over that by LCM.

B. Testing solution of Transportation Problem for Optimality

To test the optimality of a solution (Basic Feasible one or Improved one) of Transportation Problem, two methods are used. These are -

1. Modified Distribution (MODI) Method
2. Stepping Stone Method

In both the methods the approach is to test each unallocated cell of the Transportation Matrix one at a time by computing the change in cost and subsequently checking inclusion of any unoccupied cell can decrease the Total Cost of Transportation compared to that obtained in the previous solution. In fact the cell with highest negative cost change value is taken to be the one which is going to give further improvement in the solution. This procedure is continued until minimum Cost is obtained.

In practice Modified Distribution Method finds wider use compared to Stepping Stone Method due to the fact that the process of evaluating each unoccupied cell for further possible improvement is more efficient in it.

Modified Distribution method (MODI)

Following are the steps involved in Modified Distribution Method for testing optimality of a solution or for finding optimum solution of a Transportation problem.

1. For getting an optimum solution of a Transportation Problem, find the Basic Feasible Solution. In general there will be (m + n – 1) number of cell allocations in the solution (here, m = No. of Rows & n = No. of Columns of the Transportation matrix).
2. Determine the values of Row Numbers (u_i) and Column Numbers (v_j) using the relation $C_{ij} = u_i + v_j$ for each of the **allocated cells** (where C_{ij} = Unit transportation cost for allocated cell). It can be mentioned that i = m & j = n causing a total of (m + n) number of Row and Column Numbers which cannot be find by solving (m + n – 1) numbers of equations. So one of the numbers is to be chosen arbitrarily. A rule of thumb is to choose

a zero value for the number corresponding to that row or column which is having maximum number of cell allocations.

3. Compute the Opportunity Cost (Improvement Index) using the relation $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for each of the **unallocated cells.**
4. Check the sign of each Opportunity Cost. If all the **Opportunity Cost values are non-negative then the solution is optimal.** If there exists one or more opportunity cost value with negative sign then go to next step.
5. Select the unallocated cell with highest negative Opportunity Cost value as the cell to be included in the next solution.
6. Draw a closed path or loop starting from the cell selected in the previous step and going straight (either horizontally or vertically) in that direction where allocated cells are situated. Once an allocated cell is met, take a right angle turn to move straight towards another allocated cell and again take a right angle turn as before. This has to be continued until the starting cell is reached again.
7. Assign alternate plus (+) and minus (-) signs to the cells at the corner points of the loop, with a plus sign at the starting cell.
8. Of the cells with minus (-) signs, consider the one having lowest allocation. Subtract this amount of allocation from the cells with minus (-) sign and add the same to the cells with plus (+) sign. As a result the unallocated cell from where the loop started will become allocated and the cell with minus sign and having lowest allocation will become unallocated.
9. Repeat the whole procedure until all the Opportunity Cost values become non-negative i.e an Optimum Solution is obtained.

Important points:

1. Conditions to be satisfied by the Initial Feasible solution –
   - Each and every Row & Column must have at least one allocation
   - All the allocations must be made in independent cells and they should be $(m + n - 1)$ in number. It can be mentioned that the cells which make a closed loop are the Dependent cells. Others are the Independent Cells.
2. Case of equal maximum Opportunity Cost. – While revising a given Transportation solution, if two or more unallocated cells have the same largest negative Opportunity Cost, the unallocated cell by including which the decrease in the Transportation Cost is maximum should be chosen.

Illustration 4

Test the Basic Feasible Solution obtained by VAM in the previous Illustration for Optimality. In case the solution is non-optimal, find the optimum solution to get the minimum Total Cost of Transportation.

Solution:

The Basic Feasible Solution by VAM in the previous Illustration is considered and the Row Nos. (u_i) and Column Nos. (v_j) are calculated as follows –

1. The Transportation problem is having a (3×4) Matrix. So $m = 3$ and $n = 4$ here. Thus, there are three Row Nos. (u_1, u_2 and u_3) and four Column Nos. (v_1, v_2, v_3 and v_4) i.e in total 7 Nos. are to be calculated.
2. To find the Nos. we have to use the relation $C_{ij} = u_i + v_j$ for the ALLOCATED cells, but there are only 6 Allocated cells in the solution. In other words, 6 equations of the above type can be formed to find 7 unknown numbers. This is not possible by using general mathematical concepts. So the value of one of the numbers should be assumed.

3. We assume $u_2 = 0$. This is done because 2nd Row is having maximum number of allocations.
4. The calculations are as follows:

Allocated Cell	Cost (C_{ij})	$C_{ij} = u_i + v_j$	Calculated No.
P_2R_2	$C_{22} = 5$	$C_{22} = u_2 + v_2$ Or, $5 = 0 + v_2$	$v_2 = 5$
P_2R_3	$C_{23} = 8$	$C_{23} = u_2 + v_3$ Or, $8 = 0 + v_3$	$v_3 = 8$
P_2R_4	$C_{24} = 2$	$C_{24} = u_2 + v_4$ Or, $2 = 0 + v_4$	$v_4 = 2$
P_3R_4	$C_{34} = 2$	$C_{34} = u_3 + v_4$ Or, $2 = u_3 + 2$	$u_3 = 0$
P_1R_3	$C_{13} = 7$	$C_{13} = u_1 + v_3$ Or, $7 = u_1 + 8$	$u_1 = -1$
P_1R_1	$C_{11} = 3$	$C_{11} = u_1 + v_1$ Or, $3 = -1 + v_1$	$v_1 = 4$

The results are entered into the table below to show the results at a glance.

from Plants	to Retail Shops				Supply	Row Nos.
	R_1	R_2	R_3	R_4		
P_1	3 (20)	5	7 (30)	6	50	$u_1 = -1$
P_2	2	5 (20)	8 (20)	2 (35)	75	Let $u_2 = 0$
P_3	3	6	9	2 (25)	25	$u_3 = 0$
Demand	20	20	50	60		
Column No	$v_1 = 4$	$v_2 = 5$	$v_3 = 8$	$v_4 = 2$		

For the UNALLOCATED Cells, Opportunity Cost values are calculated using the relation $\Delta_{ij} = C_{ij} - (u_i + v_j)$. The calculations and the results are shown in the table below.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No.(v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) – [(3) + (4)]
P_1R_2	$C_{12} = 5$	$u_1 = -1$	$v_2 = 5$	1
P_1R_4	$C_{14} = 6$	$u_1 = -1$	$v_4 = 2$	5
P_2R_1	$C_{21} = 2$	$u_2 = 0$	$v_1 = 4$	- 2
P_3R_1	$C_{31} = 3$	$u_3 = 0$	$v_1 = 4$	- 1
P_3R_2	$C_{32} = 6$	$u_3 = 0$	$v_2 = 5$	1
P_3R_3	$C_{33} = 9$	$u_3 = 0$	$v_3 = 8$	1

So there exists two cells with negative Opportunity Cost. Hence the solution is **non-optimal**.

Highest negative Opportunity Cost appears at the cell P_2R_1. Starting from here a closed loop (as shown in the table below) is drawn and alternately (+) and (-) signs are assigned to the corners of the loop which starts with a (+).

from Plants	to Retail Shops				Supply	Row Nos.
	R_1	R_2	R_3	R_4		
P_1	(–) 20 / 3	5	(+) 30 / 7	6	50	$u_1 = -1$
P_2	(+) 2	20 / 5	(–) 20 / 8	35 / 2	75	Let $u_2 = 0$
P_3	3	6	9	25 / 2	25	$u_3 = 0$
Demand	20	20	50	60		
Column No	$v_1 = 4$	$v_2 = 5$	$v_3 = 8$	$v_4 = 2$		

Allocated quantities of the (-) signed cells are both 20. This amount is subtracted from the quantities of the (-) signed cells and added to the quantities of the (+) signed cells. As a result both the (-) signed cells become unallocated. This turns into a five cell allocated solution which means a **Degenerate solution**. To take care of this an infinitely small allocation of amount (ε) is made to the cell having higher cost between the currently vanished cells i.e. the cell P_2R_3. The new set of allocations is given in the table below along with the new set of Row and Column Nos.

from Plants	to Retail Shops				Supply	Row Nos.
	R_1	R_2	R_3	R_4		
P_1	3	5	50 / 7	6	50	$u_1 = -1$
P_2	20 / 2	20 / 5	ε / 8	35 / 2	75	Let $u_2 = 0$
P_3	3	6	9	25 / 2	25	$u_3 = 0$
Demand	20	20	50	60		
Column No	$v_1 = 2$	$v_2 = 5$	$v_3 = 8$	$v_4 = 2$		

For the UNALLOCATED Cells, again the Opportunity Cost values are calculated using the relation given before i.e. $\Delta_{ij} = C_{ij} - (u_i + v_j)$. Results are shown below.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No.(v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) – [(3) + (4)]
P_1R_2	$C_{12} = 5$	$u_1 = -1$	$v_2 = 5$	1
P_1R_4	$C_{14} = 6$	$u_1 = -1$	$v_4 = 2$	5
P_1R_1	$C_{11} = 3$	$u_1 = -1$	$v_1 = 2$	2
P_3R_1	$C_{31} = 3$	$u_3 = 0$	$v_1 = 2$	1
P_3R_2	$C_{32} = 6$	$u_3 = 0$	$v_2 = 5$	1
P_3R_3	$C_{33} = 9$	$u_3 = 0$	$v_3 = 8$	1

As no negative Opportunity Cost values are there, the solution is **Optimal.** The Optimum solution is given below.

from Plant	to Retail Shop	Shipping cost per Gallon (₹ '000)	Quantity allocated (Gallons)	Cost (₹ '000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P_1	R_3	7	50	350
P_2	R_1	2	20	40
P_2	R_2	5	20	100
P_2	R_4	2	35	70
P_2	R_3	8	$\varepsilon = 0$	0
P_3	R_4	2	25	50
TOTAL				610

So minimum Total Cost of Transportation = ₹ 610,000/-

Special cases of Transportation

Various special cases of Transportation problem are as follows –

- Unbalanced problem
- Maximization problem
- Problems with Degeneracy
- Problems having Multiple Optimum Solution
- Problems having Prohibited or Preferred Routes

1. Unbalanced problem of Transportation

One of the assumptions for applying Transportation algorithm is Total Supply capacity of the Origins is equal to the Total Demand of the Destinations. But for all practical purposes, it is seen to be met very rarely. Mostly the Supply and Demand figures are unequal. **Such a problem of Transportation is called Unbalanced problem of Transportation**.

There can be two types of Unbalanced problem – (1) Supply exceeds Demand & (2) Demand exceeds Supply. In both the cases first of all the problem has to be converted to a Balanced one and then the usual procedure of solving the problem using Transportation algorithm has to be followed.

When **Supply exceeds Demand** then the problem is converted to a Balanced one by bringing a Dummy Destination (or Column) having Demand equal to the difference between the Supply and Demand of the original problem.

When **Demand exceeds Supply** then the conversion of Unbalanced problem to a Balanced one is done by bringing Dummy Origin (or Row) with Supply capacity equal to the difference between the Supply and Demand of the original problem.

Cost of Transportation corresponding to the cells of DUMMY row or column is taken as ZERO.

Illustration 5

The products of two Plants A and B are to be transported to three Warehouses W_1, W_2 and W_3. The costs (₹ '00) of transportation of each unit from Plants to the Warehouses are indicated in the table below. Also provided are the Supply Capacities of the Plants and the Demands of the three Warehouses.

	Warehouse W_1	Warehouse W_2	Warehouse W_3	Supply Capacity
Plant A	25	17	25	300
Plant B	15	10	18	500
Demand	300	300	500	1100 \ 800

Find the Optimum Distribution Schedule and associated Cost of Transportation.

Solution:

As per the given data, Total Supply Capacity = 800 units and Total Demand = 1100 units. So the problem is unbalanced. To convert it into a balanced one, a Dummy Plant C having Supply Capacity = 1100 – 800 = 300 units is introduced. Cost of transportation from this plant to any of the warehouses is considered to be 0. The new matrix is shown below.

	Warehouse W_1	Warehouse W_2	Warehouse W_3	Supply Capacity
Plant A	25	17	25	300
Plant B	15	10	18	500
Plant C	0	0	0	300
Demand	300	300	500	1100 \ 1100

Basic Feasible Solution of the problem by VAM is shown in the table below along with the values of Row and Column Numbers (for which calculations are shown below)

Table showing Basic Feasible Solution by VAM (Optimal Solution)

Plants	Warehouse			Supply	Row Penalty			Row Nos. (u_i)
	W_1	W_2	W_3		1	2	3	
A	25	17 (100)	25 (200)	~~200~~ ~~300~~	8	8	8	$u_1 = 0$ (let)
B	15 (300)	10 (200)	18	~~200~~ ~~500~~	5	5	8*	$u_2 = -7$
C	0	0	0 (300)	~~300~~	0	-	-	$u_3 = -25$
Demand	~~300~~	~~100~~ ~~300~~	~~200~~ ~~500~~	1100				
Column Penalty	1 15	10	18*					
	2 10*	7	7					
	3 -	7	7					
Column Nos. (v_j)	$v_1 = 22$	$v_2 = 17$	$v_3 = 25$					

For the Transportation matrix, No. of Rows = m = 3 and No. of Columns = n = 3. So (m + n – 1) = 3 + 3 – 1 = 5.

In the above solution No. of Allocated Cells = 5 = m + n – 1

So the solution is Non – degenerate.

Calculation of Row Numbers (u_i) and Column Numbers (v_j) are shown in the table below.

Allocated Cell	Cost (C_{ij})	$C_{ij} = u_i + v_j$	Calculated No.
A - W_2	$C_{12} = 17$	$C_{12} = u_1 + v_2$ Or, $17 = 0 + v_2$	$v_2 = 17$
A – W_3	$C_{13} = 25$	$C_{13} = u_1 + v_3$ Or, $25 = 0 + v_3$	$v_3 = 25$
B – W_2	$C_{22} = 10$	$C_{22} = u_2 + v_2$ Or, $10 = u_2 + 17$	$u_2 = -7$
B – W_1	$C_{21} = 15$	$C_{21} = u_2 + v_1$ Or, $15 = -7 + v_1$	$v_1 = 22$
C – W_3	$C_{33} = 0$	$C_{33} = u_3 + v_3$ Or, $0 = u_3 + 25$	$u_3 = -25$

To test optimality of the solution, Opportunity Costs for each of the Unallocated Cell is computed and shown in the Table below.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No.(v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) – [(3) + (4)]
A – W_1	$C_{11} = 25$	$u_1 = 0$	$v_1 = 22$	3
B – W_3	$C_{23} = 18$	$u_2 = -7$	$v_3 = 25$	0
C – W_1	$C_{31} = 0$	$u_3 = -25$	$v_1 = 22$	3
C – W_2	$C_{32} = 0$	$u_3 = -25$	$v_2 = 17$	8

As no negative Opportunity Cost values are there, the solution is **Optimal.** The Optimum Transportation schedule along with its associated Cost is given below

from Plant	to Warehouse	Transportation cost per unit (₹ '00)	Quantity allocated (Units)	Cost (₹ '00)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
A	W_2	17	100	1700
A	W_3	25	200	5000
B	W_2	10	200	2000
B	W_1	15	300	4500
C	W_3	0	300	0
TOTAL				13200

So the minimum Total Cost of Transportation = ₹ 1,320,000

From the above solution it can be concluded that 300 units of Demand of W_3 remains unfulfilled because it is being supplied by C which is a Dummy Plant i.e. non-existent.

2. Maximization problem of Transportation

Transportation algorithm is basically meant for solving minimization problems, but the same can be successfully used for solving maximization problems, too. For that first of all, the original problem has to be converted to a minimization problem. This is done by subtracting all the elements of the given profit matrix from the highest element of the matrix. Thus we get the value of Relative Loss or Opportunity Loss against each of the given profit figures. Once the Profit Matrix given is converted to Relative Loss Matrix, the problem can be solved as a minimization problem by following usual method of solving Transportation problems.

Illustration 6

A multi plant company has three manufacturing plants M_1, M_2 and M_3. The company is dealing with a unique product and enjoy monopoly as far as competition is concerned. They have two fixed customers A and B who procures all the items produced by the company.

Cost of Production (₹ per piece of the product) of the Pants M_1, M_2 and M_3 are respectively 1500, 1600 & 1700. Selling prices to the customers A and B are ₹ 4400 and ₹ 4700 per piece respectively.

Production Capacities of the three Plants, Demands of the two Customers and the Costs of Transportation per unit from various Plants to the different Customer's premises are given as follows:

	Plant M_1	Plant M_2	Plant M_3	Demand
Customer A	₹1000	₹2000	₹1500	3500 units
Customer B	₹1500	₹3000	₹2500	3600 units
Production Capacity	2000 units	3000 units	4000 units	

Formulate the problem as LP Model. Also find the optimum solution using Transportation algorithm.

Solution:

This is a problem of Profit maximization. Profit figures per piece are computed using the relation given below.

Profit = Selling Price – (Production Cost + Transportation Cost)

Profit figures corresponding to various combinations of Customer and Plant are provided in the matrix below.

	Customer A	Customer B	Production Capacity
Plant M_1	4400 – (1500+1000) = ₹1900	4700 – (1500 + 1500) = ₹1700	2000 units
Plant M_2	4400 – (1600+2000) = ₹800	4700 – (1600 + 3000) = ₹100	3000 units
Plant M_3	4400 – (1700+1500) = ₹1200	4700 – (1700 + 2500) = ₹500	4000 units
Demand	3500 units	3600 units	7100 units \ 9000 units

From above it is clear that the given problem is an unbalanced one. Here total Production Capacity (9000 units) is more than the total Demand (7100 units) by an amount of 9000 – 7100 = 1900 units. To make it balanced, a **Dummy** Customer C having Demand of 1900 units is introduced. Profit figures for it corresponding to different Plants are each zero. The balanced Profit Matrix is given hereunder.

Profit Matrix after making the problem Balanced

	Customer A	Customer B	Customer C	Production Capacity
Plant M_1	₹ 1900	₹ 1700	0	2000 units
Plant M_2	₹ 800	₹ 100	0	3000 units
Plant M_3	₹ 1200	₹ 500	0	4000 units
Demand	3500 units	3600 units	1900 units	9000 units

Let us consider the Decision Variables x_{ij} (i = 1, 2 & 3 and j = 1, 2 & 3) as the quantities to be shipped from the Plant M_i to the Customer j. So the LPP can be written as –

Maximize $Z = 1900\, x_{11} + 1700\, x_{12} + 800\, x_{21} + 100\, x_{22} + 1200\, x_{31} + 500\, x_{32}$

Subject to the constraints:

$x_{11} + x_{12} + x_{13} = 2000$, $x_{21} + x_{22} + x_{23} = 3000$, $x_{31} + x_{32} + x_{33} = 4000$ (Row constraints or Production Capacity constraints)

$x_{11} + x_{21} + x_{31} = 3500$, $x_{12} + x_{22} + x_{32} = 3600$, $x_{13} + x_{23} + x_{33} = 1900$ (Column Constraints or Demand Constraints)

$x_{ij} \geq 0$ for all i, j = 1, 2, 3

To facilitate solution of the problem by using the algorithm of Transportation, above matrix is converted into a Relative Loss matrix by subtracting each of its elements from the highest element i.e 1900.

Relative Loss Matrix

	Customer A	Customer B	Customer C	Production Capacity
Plant M_1	0	200	1900	2000 units
Plant M_2	1100	1800	1900	3000 units
Plant M_3	700	1400	1900	4000 units
Demand	3500 units	3600 units	1900 units	9000 units

To make the calculations simpler, each figure of the above table is divided by 100. The resultant Matrix is given as follows -

	Customer A	Customer B	Customer C	Production Capacity in ₹ '00 units
Plant M_1	0	2	19	20
Plant M_2	11	18	19	30
Plant M_3	7	14	19	40
Demand in 00 units	35	36	19	90

Basic Feasible solution of the problem by VAM is shown in the Table below along with the Row and Column Nos.

From \ To Plant	Customer A	Customer B	Customer C	Production Capacity	Row Penalty 1	Row Penalty 2	Row Penalty 3	Row Nos. (u_i)
M_1	0	2 (20)	19	~~20~~	2	-	-	$u_1 = 2$
M_2	11	18 (11)	19 (19)	~~11~~ ~~30~~	7	7	1	$u_2 = 18$
M_3	7 (35)	14 (5)	19	~~5~~ ~~40~~	7	7*	5*	$u_3 = 14$
Demand	~~35~~	~~16~~ ~~5~~ ~~36~~	~~19~~	90				
Column Penalty 1	7	12*	0					
Column Penalty 2	4	4	0					
Column Penalty 3	-	4	0					
Column Nos. (v_j)	$v_1 = -7$	$v_2 = 0$ (let)	$v_3 = 1$					

For the Transportation matrix, No. of Rows = m = 3 and No. of Columns = n = 3. So (m + n – 1) = 3 + 3 – 1 = 5.

In the above solution No. of Allocated Cells = 5 = m + n – 1

So the solution is Non – degenerate.

Calculation of Row Numbers (u_i) and Column Numbers (v_j) are shown in the table below.

Allocated Cell	Cost (C_{ij})	$C_{ij} = u_i + v_j$	Calculated No.
M_1 - B	$C_{12} = 2$	$C_{12} = u_1 + v_2$ Or, $2 = u_1 + 0$	$u_1 = 2$
M_2 - B	$C_{22} = 18$	$C_{22} = u_2 + v_2$ Or, $18 = u_2 + 0$	$u_2 = 18$
M_2 - C	$C_{23} = 19$	$C_{23} = u_2 + v_3$ Or, $19 = 18 + v_3$	$v_3 = 1$
M_3 - B	$C_{32} = 14$	$C_{32} = u_3 + v_2$ Or, $14 = u_3 + 0$	$u_3 = 14$
M_3 - A	$C_{31} = 7$	$C_{31} = u_3 + v_1$ Or, $7 = 14 + v_1$	$v_1 = -7$

To test optimality of the solution, Opportunity Costs for each of the Unallocated Cell is computed and shown in the following Table.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No.(v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) – [(3) + (4)]
M_1 - A	$C_{11} = 0$	$u_1 = 2$	$v_1 = -7$	5
M_1 - C	$C_{33} = 19$	$u_1 = 2$	$v_3 = 1$	16
M_2 - A	$C_{21} = 11$	$u_2 = 18$	$v_1 = -7$	0
M_3 - C	$C_{33} = 19$	$u_3 = 14$	$v_3 = 1$	4

As no negative Opportunity Cost values are there, the solution is **Optimal.**

The Optimum Transportation schedule along with the maximum total profit is given hereunder.

from Plant	to Customer	Profit per unit (₹)	Quantity allocated (₹ '00 Units)	Profit (₹)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
M_1	B	1700	20	34000
M_2	B	100	11	1100
M_2	C	0	19	0
M_3	A	1200	35	42000
M_3	B	500	5	2500
TOTAL				79600

Maximum Total Profit = ₹ 79,60,000 (After considering proper unit of the Quantity allocated values)

[**N.B** – Profit figures are taken from the PROFIT MATRIX prepared earlier. Plant M_2 can produce 1900 units less because these many units produced by it are allocated to the Customer C which is actually a DUMMY Customer]

3. Problems with Degeneracy

A Transportation problem with (m × n) Cost Matrix should have (m + n – 1) Numbers of Cell allocations at any stage of solution i.e either in Basic Feasible Solution or in any Improved Solution. In case the Numbers of allocations are less than (m + n – 1), the solution is said to be Degenerate. In other words Degeneracy exists in such a solution.

Degeneracy can be observed in two different stages while solving a Transportation problem –

- At the stage of obtaining Initial Feasible solution
- During the stage of improving a solution

Degeneracy at the stage of Initial Feasible solution

To resolve such a situation of Degeneracy, an infinitely small quantity [denoted by Greek letter 'Epsilon' (ε)] is introduced into the solution to one or more of the unoccupied cells so that the number of cell allocations become (m + n – 1). For all practical purposes the value of ε is taken to be ZERO. It is introduced to **the least cost unoccupied independent cell** of the matrix.

Degeneracy during improvement of a solution

This kind of situation occurs when the inclusion of the unoccupied cell with maximum negative Opportunity Cost results in vacating two or more occupied cells simultaneously. To resolve such a situation of Degeneracy, allocation of an infinitely small quantity ε **is done to one or more of the recently vacated cells** so that the number of allocated cells become (m + n – 1).

[This kind of situation of Degeneracy has already been encountered in Illustration 4 of this Module]

Illustration 7

A manufacturing company has three Pants X, Y and Z which supply to the Distributors located at A, B, C, D and E. Monthly production capacities of the Plants are respectively 80, 50 and 90 units. Monthly requirement of the Distributors are 40, 40, 50, 40 and 80 units respectively. Unit Transportation Costs (₹) are given below.

from Plants	to Distributors				
	A	B	C	D	E
X	5	8	6	6	3
Y	4	7	7	6	6
Z	8	4	6	6	3

Determine the Optimum Schedule of distribution of the company in order to minimize the Total Cost of Transportation.

Solution:

As per the given information – Total Production Capacity of the three Plants = 80 + 50 + 90 = 220 Units and Total Requirement of the five Distributors = 40 + 40 + 50 + 40 + 80 = 250 Units

Demand > Supply here. So this is an Unbalanced problem. To make it Balanced, we introduce a Dummy Plant having Production Capacity = 250 – 220 = 30 Units.

Unit Transportation Cost from this Dummy Plant to any of the Distributors is taken as ZERO.

Now the Basic Feasible solution of the problem is obtained using VAM and shown in the Table follows.

Table showing Basic Feasible Solution by VAM

From \ To	A	B	C	D	E	Production Capacity	Row Penalty 1	2	3	4	5	6
X	5	8	6 (50)	6	3 (30)	~~80~~ ~~50~~	2	2	2	2	1	0
Y	4 (40)	7	7	6 (10)	6	~~50~~ ~~10~~	2	2	2	2^*	2^*	1
Z	8	4 (40)	6	6	3 (50)	~~90~~ ~~50~~	1	1	3^*	-	-	-
Dummy	0	0	0	0 (30)	0	~~30~~	0	-	-	-	-	-
Requirement	~~40~~	~~40~~	~~50~~	~~10~~ ~~40~~	~~30~~ ~~80~~	250						
Column Penalty 1	4	4	6	6^*	3							
2	1	3^*	0	0	0							
3	1	-	0	0	0							
4	1	-	1	0	3^*							
5	1	-	1	0	-							
6	-	-	1	0	-							

For the Transportation matrix, No. of Rows = m = 4 and No. of Columns = n = 5. So (m + n – 1) = 4 + 5 – 1 = 8.

In the above solution No. of Allocated Cells = 7 ≠ m + n – 1

So the solution is **Degenerate.**

To resolve Degeneracy we have to allocate an infinitely small quantity ε to the least cost unoccupied cell which should be independent. Here 0 is the least cost figure among all the unoccupied cells and it occurs at four places namely (Dummy – A), (Dummy – B), (Dummy – C) & (Dummy – E). Of these, (Dummy – A) is not an Independent Cell because starting from here a loop can be formed using this cell & the cells (Y – A), (Y – D) & (Dummy – D).

So we are left with the cells (Dummy – B), (Dummy – C) and (Dummy – E). Arbitrarily we choose the cell Dummy – C and allocate the infinitely small quantity ε. Subsequently Row and Column Numbers are calculated as follows:

Allocated Cell	Cost (C_{ij})	$C_{ij} = u_i + v_j$	Calculated No.
X - C	$C_{13} = 6$	$C_{13} = u_1 + v_3$ Or, $6 = 0 + v_3$	$v_3 = 6$
X - E	$C_{15} = 3$	$C_{15} = u_1 + v_5$ Or, $3 = 0 + v_5$	$v_5 = 3$
Z - E	$C_{35} = 3$	$C_{35} = u_3 + v_5$ Or, $3 = u_3 + 3$	$u_3 = 0$
Z - B	$C_{32} = 4$	$C_{32} = u_3 + v_2$ Or, $4 = 0 + v_2$	$v_2 = 4$
Dummy - C	$C_{43} = 0$	$C_{43} = u_4 + v_3$ Or, $0 = u_4 + 6$	$u_4 = - 6$
Dummy - D	$C_{44} = 0$	$C_{44} = u_4 + v_4$ Or, $0 = - 6 + v_4$	$v_4 = 6$
Y - D	$C_{24} = 6$	$C_{24} = u_2 + v_4$ Or, $6 = u_2 + 6$	$u_2 = 0$
Y - A	$C_{21} = 4$	$C_{21} = u_2 + v_1$ Or, $4 = 0 + v_1$	$v_1 = 4$

All the cell allocations along with the respective Row and Column Numbers are given in the Table below:

Table showing Basic Feasible Solution by VAM after resolving Degeneracy

From \ To	A	B	C	D	E	Production Capacity	Row Nos. (u_i)
X	5	8	6 (50)	6	3 (30)	80	$u_1 = 0$ (let)
Y	4 (40)	7	7	6 (10)	6	50	$u_2 = 0$
Z	8	4 (40)	6	6	3 (50)	90	$u_3 = 0$
Dummy	0	0	0 (e)	0 (30)	0	30	$u_4 = - 6$
Requirement	40	40	50	40	80		
Column Nos. (v_j)	$v_1 = 4$	$v_2 = 4$	$v_3 = 6$	$v_4 = 6$	$v_5 = 3$		

To test optimality of the solution, Opportunity Costs for each of the Unallocated Cell is computed and shown in the following Table.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No.(v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) – [(3) + (4)]
X - A	$C_{11} = 5$	$u_1 = 0$	$v_1 = 4$	1
X - B	$C_{12} = 8$	$u_1 = 0$	$v_2 = 4$	4
X - D	$C_{14} = 6$	$u_1 = 0$	$v_4 = 6$	0
Y - B	$C_{22} = 7$	$u_2 = 0$	$v_2 = 4$	3
Y - C	$C_{23} = 7$	$u_2 = 0$	$v_3 = 6$	1
Y - E	$C_{25} = 6$	$u_2 = 0$	$v_5 = 3$	3
Z - A	$C_{31} = 8$	$u_3 = 0$	$v_1 = 4$	4
Z - C	$C_{33} = 6$	$u_3 = 0$	$v_3 = 6$	0
Z - D	$C_{34} = 6$	$u_3 = 0$	$v_4 = 6$	0
Dummy - A	$C_{41} = 0$	$u_4 = -6$	$v_1 = 4$	2
Dummy - B	$C_{42} = 0$	$u_4 = -6$	$v_2 = 4$	2
Dummy - E	$C_{45} = 0$	$u_4 = -6$	$v_5 = 3$	3

As no negative Opportunity Cost values are there, the solution is **Optimal**

The Optimum Transportation Schedule along with the associated Total Cost is given below:

from Plant	to Distributor	Transportation cost per unit (₹)	Quantity allocated (Units)	Cost (₹)
(1)	(2)	(3)	(4)	(5) = (3) X (4)
X	C	6	50	300
X	E	3	30	90
Y	A	4	40	160
Y	D	6	10	60
Z	B	4	40	160
Z	E	3	50	150
Dummy	D	0	30	0
TOTAL				920

So the minimum Total Cost of Transportation = ₹ 920.

From the above solution it can be concluded that 30 units of Demand of Distributor 'D' remains unfulfilled because it is being supplied by the Dummy Plant which is non-existent.

4. Transportation problems having Multiple Optimum Solution

Multiple Optimum Solution of a Transportation problem means the problem is having more than one optimum solution. In other words, the minimum Total Cost of Transportation for all such solutions are same but the routes of transportation are different.

A Transportation problem is said to have Multiple Optimum Solution if there exist one or more unoccupied cell in the final Table with ZERO Opportunity Cost.

If such situation occurs for a problem then to get another optimum solution a loop has to be formed starting from the cell (or the cells). This will give a new set of allocations without disturbing the minimum Total Transportation Cost. In all of the previous Illustrations 5, 6 and 7 such situation is there i.e. ZERO Opportunity Cost cells are there in the Optimum solution obtained in the answer.

Illustration 8

Find an Alternative Solution to the Transportation Problem of Illustration 5.

Solution:

Optimum solution already obtained in Illustration 5 shows Opportunity Cost value = 0 for the cell ($B – W_3$). So a loop beginning from ($B – W_3$) is drawn and shown in the table below.

Basic Feasible Solution by VAM (Optimal)

From \ To	Warehouse			Supply
	W_1	W_2	W_3	
A	25	17 (+) 100	25 200 (–)	300
B	15 300	10 200 (–)	18 (+)	500
C	0	0	0 300	300
Demand	300	300	500	1100

From the drawn loop it is clear that the allocations with (-) sign are both 200. So this quantity is to be subtracted from the allocations of the cells with (-) sign and added to the quantities of the cells with (+) sign. The resultant allocations are shown in the Table below.

Alternative Optimum Solution

From \ To	Warehouse W$_1$	Warehouse W$_2$	Warehouse W$_3$	Supply
A	25	17 (300)	25	300
B	15 (300)	25	18 (200)	500
C	0	0	0 (300)	300
Demand	300	300	500	1100

The Transportation schedule and its associated Total Cost is represented as follows.

from Plant	to Warehouse	Transportation cost per unit (₹ '00)	Quantity allocated (Units)	Cost (₹ '00)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
A	W_2	17	300	5100
B	W_1	15	300	4500
B	W_3	18	200	3600
C	W_3	0	300	0
TOTAL				13200

So Total Cost of Transportation = ₹ 13200 which is same as the Total Cost obtained in Illustration 5. Thus the Transportation schedule though different yields same minimum Total Cost.

[The solution above is Degenerate because No. of Cell Allocations = 4 m + n – 1 (5), but this being the Optimum Solution is not required to be resolved]

5. Transportation problems with Prohibited or Preferred Routes

Some times in practical life one may have to come across situations of restriction or prohibition of certain routes for Transportation of goods. The reason behind this is situations occurred due to Natural Calamities (like Floods, Earthquake etc.) or due to War, Strike called by the political parties etc.

Such restrictions are handled in the Transportation problem by assigning a very high cost (Say M or ∞) in the prohibited routes so that these routes are never included in the solution. The usual method of solving a Transportation problem is used.

EXERCISE

A. Theoretical Questions:

Multiple Choice Questions

1. Which of the following considers difference between least cost and the cost just before least for each row and column while finding Basic Feasible Solution in Transportation?
   (a) North West Corner Method
   (b) Least Cost Method
   (c) Vogel's Approximation Method
   (d) Both (b) and (c) above

2. When the total allocation of a Transportation Problem match with supply and demand values, the solution is –
   (a) Non-degenerate
   (b) Feasible
   (c) Degenerate
   (d) None of the above

3. The solution to a Transportation Problem with 'm' sources ad 'n' destinations is feasible if the number of cell allocations are –
   (a) m + n
   (b) mn
   (c) m – n -1
   (d) m + n – 1

4. To resolve Degeneracy in the solution of a Transportation Problem an infinitely small allocation is made to the solution already obtained. This allocation is known as –
   (a) Dummy
   (b) Epsilon
   (c) ε – the Greek letter
   (d) All of the above except (a)

5. Which of the following is not correct with respect to Transportation as a tool of Quantitative Technique?
   (a) Transportation technique is a special case of LP.
   (b) Transportation technique might give rise to solutions which are degenerate.
   (c) No Transportation problem can be given with supply ≠ demand.
   (d) Using Transportation technique one can maximize an Objective Function.

6. Which of the following method is used to test optimality of a solution in Transportation?
   (a) Modified Distribution
   (b) Simplex

(c) VAM

(d) LCM

7. In a solution of Transportation problem, empty cells are called –

(a) Unoccupied cells

(b) Unallocated cells

(c) Empty cells

(d) All of the above

8. The Transportation Problem deals with the transportation of –

(a) Single product from a source to several destinations

(b) Several products from a source to a destination.

(c) Single product from several sources to a destination.

(d) Single product from several sources to several destinations.

9. In NWCM, first allocation is made at –

(a) Upper left hand corner of the table.

(b) Lower right hand corner of the table.

(c) Upper right hand corner of the table.

(d) Lower left hand corner of the table.

10. One of the disadvantages of North West Corner rule for finding Initial Feasible Solution of Transportation problem is –

(a) It is complicated to use

(b) It leads to non-optimal solution

(c) It does not take into account unit cost of transportation.

(d) Generally it provides degenerate solution.

11. When total demand and supply are equal then the Transportation problem is said to be –

(a) A problem having multiple optimum solutions.

(b) A problem having degeneracy.

(c) A balanced one.

(d) None of the above.

12. Which one of the following is correct?

(a) The dummy source or destination is used in a Transportation problem to resolve degeneracy.

(b) The dummy source or destination is used in a Transportation problem to make it balanced.

(c) The dummy source or destination is used in a Transportation problem to ensure its cost effectiveness.

(d) All the above statements are correct.

13. For solving a maximization problem by Transportation algorithm, the very first step is to –
    (a) Subtract smallest cost element of the matrix from all the other cost elements.
    (b) Subtract all the cost elements of the matrix from the highest element of the same.
    (c) Add smallest cost element of the matrix to all the other cost elements.
    (d) Add highest cost element of the matrix to all the other elements.

14. Which of the following methods is used for finding an initial feasible solution of a Transportation Problem?
    (a) Simplex
    (b) Least Cost
    (c) Hungarian
    (d) Big M

15. Which of the following is a method for improving an initial solution of a Transportation problem?
    (a) Stepping Stone
    (b) North West Corner
    (c) Intuitive Lowest Cost
    (d) All of the above

16. Basic Feasible Solution for a Transportation problem is given as follows –

To / From	Warehouse W_1	Warehouse W_2	Supply
A	5 (10)	7	10
B	8 (15)	9 (25)	40
Demand	25	25	50

Given, the Unit Transportation Costs are in Rupees. Can this solution be improved?
    (a) Yes the solution can be improved by ₹ 50
    (b) Yes the solution can be improved by ₹ 100
    (c) No the solution is optimal
    (d) Yes the solution can be improved by ₹ 10.

17. The Initial Feasible Solution of a Transportation Problem can be obtained by different methods. The only restriction is that –
    (a) The edge constraints of supply and demand are satisfied.
    (b) The solution must be obtained using VAM.

(c) The solution should be non-degenerate.

(d) All of the above.

18. The purpose of Stepping Stone Method is to –

(a) Facilitate moving from a feasible solution to an optimal solution.

(b) Test optimality of a solution.

(c) Both the two above.

(d) None of the above.

19. Which one of the following is the purpose of a dummy source or dummy destination in a Transportation Problem?

(a) To convert the problem from unbalanced to balanced.

(b) To make the solution non-degenerate.

(c) To provide a means of a dummy problem.

(d) To make sure that the total cost is not exceeding a predetermined figure.

20. An important assumption of Transportation technique is –

(a) There is only one optimal solution for each problem

(b) There are no economies of scale if huge quantities are transported from one source to one destination.

(c) The number of dummy sources and destinations must be equal.

(d) None of the above.

21. The equation $C_{ij} = u_i + v_j$ is used to calculate –

(a) An improvement index for the Stepping Stone Method.

(b) The MODI cost values u_i and v_j

(c) The Degeneracy index.

(d) None of the above

22. For an unbalanced problem of Transportation, the cost coefficients for each of the created cells is –

(a) Very high positive value

(b) Very high negative value

(c) Zero

(d) One

23. A degenerate solution of a Transportation Problem means –

(a) Total supply is not equal to the total demand.

(b) Some allocations have become negative.

(c) The obtained solution is not feasible.

(d) Both (a) and (b) but not (c).

24. Multiple optimum solutions exist for a Transportation Problem when –
    (a) There is at least one unoccupied cell of the obtained optimal solution which has zero opportunity cost.
    (b) There is unused route of Transportation having all the cells with positive opportunity cost.
    (c) There is unused route of Transportation with further scope of reducing total cost of transportation.
    (d) There is one and only one unoccupied cell of the obtained optimal solution with zero opportunity cost.

25. In an iteration while moving from one solution to the next, degeneracy occurs when –
    (a) The closed loop indicates a diagonal move.
    (b) Two or more of the allocated cells in the closed loop with minus sign have same lowest allocation.
    (c) Two or more allocated cells are on the closed loop but neither of them represent a corner of the loop.
    (d) Either one of the above

26. Left hand side of the equation $\Delta_{ij} = C_{ij} - (u_i + v_j)$ is known as –
    (a) Opportunity Cost
    (b) Improvement Index
    (c) Both (a) and (b)
    (d) None of the above

27. The highest negative opportunity cost value in an unused cell of a Transportation Matrix is chosen to improve the current solution because –
    (a) It represents maximum possible cost reduction per unit
    (b) It ensures no violation of Rim Condition.
    (c) It represents per unit cost improvement.
    (d) Either one of the above.

28. Which of the following statements is best suited to the Transportation solution given below?

From \ To	Warehouse			Supply
	W_1	W_2	W_3	
A	3 (20)	5 (50)	9	70
B	5	4 (30)	7	30
C	10 (40)	8	3 (80)	120
Demand	60	80	80	220

    (a) The solution is degenerate.

(b) The solution can be improved by shipping from C to W_2

(c) The solution can be improved by shipping from B to W_1

(d) NWCM has been used to develop the solution.

29. The Total Cost of Transportation for the Solution Matrix given in the Q. No. 28 is –

(a) ₹1070

(b) ₹1130

(c) ₹1350

(d) ₹1050

30. Which of the following statement is true in respect of the solution of a Transportation Problem?

From \ To	Warehouse W_1	Warehouse W_2	Warehouse W_3	Supply
A	25	17 (300)	25	300
B	15 (300)	10	18 (200)	500
C	0	0	0 (300)	300
Demand	300	300	500	1100

(a) The problem is an unbalanced one with Demand > Supply.

(b) Plant C is a Dummy Plant.

(c) Demand of Warehouse W_3 will not be completely fulfilled.

(d) All of the above.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	b	d	d	c	a	d	d	a	c	c	b	b	b	a
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
c	a	c	a	b	b	c	c	a	b	c	a	b	a	d

State True or False

1. The first step of solving Transportation problem is to find Basic Feasible Solution.
2. Transportation algorithm is basically meant for minimization of Objective Function. So maximization problems cannot be solved using this algorithm.

3. While finding an improved solution in course of getting the optimal Transportation solution, a closed loop is drawn through even number of cells of the matrix, of which one is unallocated and the rest are allocated.
4. Basic feasible solution of a Transportation problem by VAM always gives optimal or near optimal solution.
5. Total cost of Transportation as per the schedule of Q. No. 30 of MCQ Section above is ₹ 13000/-
6. LCM is the most popular method of carrying out Optimality test of Initial Solution of a Transportation problem.
7. Transportation technique was first developed while studying the problem of distribution of a product from several sources to numerous locations.
8. Using Graphical Method of Linear Programming to solve a problem of Transportation is very common.
9. Situation of Transportation under prohibition on usage of particular route occurs due to natural calamity.
10. Improved solution of a Transportation problem is obtained by creating a closed loop using at least four allocated cells of the existing solution.
11. For a (5 × 4) Transportation Matrix the Basic Feasible Solution is seen to have 6 allocations. The solution can be called non-degenerate.
12. Problems of Transportation with Degeneracy generally have multiple optimum solutions.
13. Basic feasible solution of a Transportation problem should have at least one allocation in each row and column of the matrix.
14. In NWCM if the resource availability of the 1st row is exhausted first then we move down the 2nd row and 1st column to make an allocation which either exhausts the resource availability of 2nd row or satisfies the remaining demand of the destination represented in 1st column.
15. Full form of LCM, as used in finding Basic Feasible Solution of a Transportation problem is Least Common Multiple.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
T	F	T	T	F	F	T	F	T	F	F	T	T	T	F

Fill in the blanks

1. In a problem of transportation with prohibited routes, the cells having restriction are considered to have unit cost of transportation equal to _____.
2. Any transportation problem can be solved as a special case of LPP using _____ method.
3. Degeneracy can occur at ___ different stages while solving a Transportation Problem.
4. By converting a given Profit Matrix to____ Matrix a problem of maximization can be solved by using Transportation algorithm.
5. To resolve Degeneracy during Initial Feasible Solution of a Transportation problem an infinitely small quantity ε is allocated to the least cost _____ unoccupied cell of the solution.
6. From the point of view of optimality, the Transportation solution given in Q. No. 30 above is _____.

7. From the point of view of degeneracy, the Transportation solution given in Q. No. 30 above is _____.
8. The objective of using Transportation algorithm is to _____ the Total Cost of Transportation.
9. In a Transportation Table an ordered set of ___ or more cells is said to form a closed loop.
10. Transportation problems are also called _____ problem after the name of its inventor F. L. Hitchcock.
11. Chance of getting optimal solution is ___ when Basic Feasible Solution is obtained by NWCM than by VAM.
12. MODI is a ____ time consuming method to test optimality of a solution than Stepping Stone Method.
13. The column number v_1 for the solution in Q. No. 16 above is _____.
14. To calculate Row and Column numbers (u_i & v_j) for a Transportation Table only the ____ cells are used.
15. Equality of Supply and Demand is a constraint of Transportation algorithm and is called ____ condition.

Answers:

1.	Infinity	**2.**	Simplex
3.	Two	**4.**	Relative or Opportunity Loss
5.	Independent	**6.**	Optimal
7.	Degenerate	**8.**	Minimize
9.	Four	**10.**	Hitchcock
11.	Less	**12.**	Less
13.	8	**14.**	Allocated
15.	Rim		

Short Essay Type Questions

1. "When formulated as LPP, constraints of a Transportation problem are always in the form of equations" – Explain the sentence written.
2. Why it takes more time to arrive at the optimal solution if the Initial Feasible Solution is obtained by North West Corner rule?
3. Define the term "Penalty" as applied to Vogel's Approximation Method.
4. Why alternately plus and minus signs are provided (with a plus sign at the starting point), to the corner points of a loop?
5. How the loop used for getting an alternative solution is different from the loop for improving an existing solution of a Transportation problem?

Essay type questions

1. Discuss on the technique of solving a maximization problem using Transportation algorithm.
2. Explain with an example how a Transportation problem can be formulated as LPP.
3. Write short notes on –
   (i) Least Cost Method of finding Initial Feasible Solution of a Transportation problem
   (ii) Use of Opportunity Cost while testing optimality of a Solution of Transportation problem
   (iii) Method of solving Transportation problem with Preferred Routes.
4. What are different stages of solution where Degeneracy can occur in a Transportation problem? Explain in brief how such situations can be resolved.
5. What is meant by Unbalanced problem of Transportation and how such problems are solved?

B. Numerical Questions:

Comprehensive Numerical Problems

1. ABC Enterprises are having three Plants, for manufacturing Dry Cells, located at three different states of the country. Production cost differs from Plant to Plant. There are four dedicated Sales Offices of the Company located at the four metro cities in the four regions of the country and a Head Office, in the capital of the country, which also takes care of sales. The Sales prices are different from one region to the other and also in the capital city. Unit Production Cost and Maximum Production Capacity of each Plant is provided in the first table below. Also provided are the data on Shipping Cost from each Plant to the different Sales Offices as well as Demand and Price at which they can sell.

   You have to find the Production and Distribution schedule which maximises the Profit of the company.

Table 1 showing Production Cost and Capacity of the Plants

Plant Number	Maximum Production Capacity	Production Cost (₹)
1	150 Units	20
2	200 Units	22
3	125 Units	18

Table 2 showing Shipping Cost per unit from different Plants to various Sales Offices

from Plants	to Sales Offices at				
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City
1	1	1	5	9	4
2	9	7	8	3	6
3	4	5	3	2	7

Table 3 showing Demand (Units) and Selling Prices (₹) for different Sales Offices

Item	for Sales Offices at				
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City
Demand	80	100	75	45	125
Selling Price	30	32	31	34	29

2. A company wishes to determine an Investment Strategy for each of the next four years. Five investment types have been selected, investment capital has been allocated for each of the coming four years and maximum investment levels have been established for each investment type. An assumption is that amounts invested in a year will remain invested until the end of the planning horizon of four years. The following table summarises the data for this problem. The values in the body of the table represent net return on investment of one rupee up to the end of the planning horizon. For example a rupee invested in investment type B at the beginning of the year 1 will grow to ₹ 1.90 by the end of the 4th year yielding a net return of ₹ 0.90.

Table showing Net Return data

Investment made at the beginning of the year	Investment Type					Amount available for Investment (₹000)
	A	B	C	D	E	
1	0.80	0.90	0.60	0.75	1.00	500
2	0.55	0.65	0.40	0.60	0.50	600
3	0.30	0.25	0.30	0.50	0.20	750
4	0.15	0.12	0.25	0.35	0.10	800
Maximum possible investment (₹ '000)	750	600	500	800	1000	

You need to determine the amount to be invested at the beginning of each year in different types of investment so as to maximize the net rupee return for the period of four years. Also find the maximum value of net return on investment.

3. Consider the problem of scheduling the weekly production of certain items for the next four weeks. The cost of production of the item is ₹10 for the first 2 weeks and ₹15 for the last 2 weeks. The weekly demands are 300, 700, 900 and 800 units, which must be met. The plant can produce maximum 700 units per week. In addition the company can employ overtime during the 2nd and 3rd week. This increases the weekly production by additional 200 units, but the production cost increases by ₹5. Excess production can be stored at a unit cost of ₹3 per week. How should the production be scheduled so as to minimize the total cost? How much production is to be carried out by overtime?

Answer:

1. Production and Distribution Schedule

from Plant	to Sales Office	Number of units allocated
1	Northern Region	50
1	Southern Region	100
2	Eastern Region	25
2	Capital City	125
3	Northern Region	30
3	Western Region	75
3	Eastern Region	20

2. Schedule of Amounts to be invested in different types of Investment in various years

Year of Investment	Investment type	Amount to be invested (₹000)
1	E	500
2	B	600
3	D	750
4	A	250
4	C	500
4	D	50

Maximum net return on investment = ₹ 14,45,000

3. [**Hints** – As per the given condition, Available options of production in different weeks can be written down as – Normal time production for week 1, Normal time production for week 2, Overtime production for week 2, Normal time production for week 3, Overtime production for week 3 and Normal time production for week 4.

   Also Cost of producing an item in Normal time for the 1st two weeks = ₹ 10. But if the same item is produced in a week and used in a subsequent week then there is Carrying Cost @ ₹ 3 per week. Hence an item produced in the 1st week and used in the 2nd week will have a cost of ₹ 13, in the 3rd week it will be ₹16 and so on. Similarly Cost of producing an item in overtime = ₹15. Thus cost of using the same item in the next week = ₹18, in the week next to that = ₹21 and so on.

   Thus the given information can be tabulated as follows –

Produced during week	Cost (₹ per unit) when used in the week					Quantity available (Nos.)
	1	2	3	4	Dummy	
1	10	13	16	19	0	700
2 (Normal)	M	10	13	16	0	700

Produced during week	Cost (₹ per unit) when used in the week					Quantity available (Nos.)
	1	2	3	4	Dummy	
2 (Overtime)	M	15	18	21	0	200
3 (Normal)	M	M	15	18	0	700
3 (Overtime)	M	M	20	23	0	200
4	M	M	M	15	0	700
Demand	300	700	900	800	500	3200

[As the production of 2nd week cannot be used in the 1st week, the allocation in the cell (2,1) is not possible. In other words it is a case of prohibited transportation. To take care of this restriction the cost figues in all such cells have been taken as M]

Now the problem can be solved using usual technique of solving Transportation problems.

The optimal schedule is given as follows -

Produced in week	Used in week	Number of units
1	1	300
1	3	200
1	4	100
1	Dummy	100
2 (Normal time)	2	700
2 (Overtime)	Dummy	200
3 (Normal time)	3	700
3 (Overtime)	Dummy	200
4	4	700

Minimum Total Cost = ₹ 36,100.

From the table above it is clear that overtime productions have been allocated only to the Dummy week. Hence no production in overtime is necessary. In fact there will be spare capacity of 100 units in the 1st week because this quantity of the production of 1st week is allocated to Dummy week.

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